

Report for SICSA of the activities of Prof. P. J. Scott, SICSA Distinguished Visitor, August 9, 2011

My SICSA activities while visiting Informatics at U. Edinburgh were divided into giving a series of public lectures, consulting informally with colleagues and postgraduate students and taking part in activities of the Scottish Informatics Community. I extended my original stay in Edinburgh, along with my SICSA schedule, to cover 6 months. A list of specific talks presented as part of this SICSA visit is appended.

During my time in Edinburgh, I worked on the following research themes:

1. *Partially Traced Categories and Feedback in Networks*

In this work, I studied the axioms of partially traced categories as well as feedback in networks in a general algebraic setting., and concrete connections with circuit theory and analog computation. This latter work is still ongoing, with discussions with Edinburgh colleagues (especially Prof. G. Plotkin).

The general program of clarifying the axioms and connections with quantum computing has been further developed during my stay. Two papers relating to this theme were finished during the visit, and a third paper begun.

(i) Partially Traced Categories, by O. Malherbe, P. Scott and P. Selinger , submitted.

(ii) Categorical traces from single-photon linear optics, by P. Hines and P. Scott , to appear in *Proc. 2008 Clifford Lectures* .

(iii) Presheaf models of quantum computation, by O. Malherbe, P. Scott, and P. Selinger (begun during this visit; in progress).

2. *MV Algebras, Duality, and AF C*-algebras*

With Dr. Mark Lawson (Heriot-Watt) we began to study the theory of MV-algebras, which are models of many-valued logics. These have intimate connections with operator algebra (so-called AF C*-algebras) and the duality theory of certain inverse semigroups studied by Dr. Lawson. In particular, we are currently working on a program to clarify the connections of certain MV-algebras, as studied by D. Mundici, with a kind of non-commutative version of Stone-Duality (actually between pseudo groups and étale groupoids) arising from recent work of M. Lawson and D. Lenz. There are many possible applications of this programme: for example, to Girard's Geometry of Interaction, to inverse semigroups of partial symmetries of algebraic structures and to frame/locale-theory in topos theory. There are known applications of MV algebras to a wide array of computational phenomena, from quantum structures to error-correcting codes and algorithmic learning theory. We hope to examine these applications for certain specific classes of MV-algebras arising from the above duality theories.

This is work in progress.

3. *Polarized Categories, Polarized Logics and Geometry of Interaction.*

Polarized logics and the theory of focussing arose in the foundations of proof search and theorem proving. It originated in linear logic (in early work of Andreoli), and has since been vastly expanded in work by D. Miller, F. Pfenning, et al. Polarized linear logic (developed by O. Laurent) has deep connections with several areas of computer science and logic, including game semantics (from work of O. Laurent and P.-A. Melliès). I

(with my colleague M. Hamano (U. Tokyo)) have been studying Girard's Geometry of Interaction (GoI) in the Polarized Setting. In this work we construct new models of polarized logics (based on certain bimodule-based pointed models on top of GoI models), and characterize focussing as an invariant of Girard's Execution formula. We are currently examining the dynamical behaviour (arising from unwinding the Execution formula) and the behaviour of program evolution in this setting, as well as constructing *compact closed* polarized models of such logics by GoI methods. This is work still in progress.

(iv) On Geometry of Interaction for Polarized Linear Logic, by M. Hamano and P. Scott (paper in progress).

List of SICSA activities:

- Lab Lunch talk: LFCS, Informatics, U. Edinburgh, February 8, 2011: Traced categories: algebraic structure of feedback and partial feedback in networks.
- 3 lecture Minicourse in LFCS (U. Edinburgh): "Traced monoidal categories and Feedback" on Friday, February 18, Tuesday, February 22, and Friday, February 25.
- Invited speaker: Scottish Categories Meeting (organized by T. Leinster (Glasgow) and N. Ghani (Strathclyde)) May 13, 2011.
- Invited speaker (University of Strathclyde CS Department) and scientific meetings with Prof. Neil Ghani, Dr. Patricia Johann, and the other researchers in Prof. Ghani's University of Strathclyde Research Group.
- Weekly meetings with Dr. Mark Lawson (Heriot-Watt) on joint projects in logic and theoretical computer science. Spent day at Heriott-Watt (Maths and CS) discussing with colleagues in Computer Science (esp. Dr. Murdoch Gabbay).
- Consulted with Professors Roy Dyckhoff and Ursula Martin (U. St. Andrews, Computer Science), on Partially Traced Categories and Network Algebra, 30 June, 2011.
- Minicourse: Lectures on the Free Topos, LFCS (Informatics), U. Edinburgh July 13, 2011.
- Invited member of the Program Committee, Trends in Functional Programming: special event: "75 years of lambda calculus" , to be held at University of St Andrews (CS Department), June, 2012.

I am planning on continuing consultation and research contacts with Scottish researchers whom I visited during this period. This includes Prof. Gordon Plotkin (Edinburgh), Prof. Neil Ghani and his research group (Strathclyde), Prof. Roy Dyckhoff (St. Andrews) and Dr. Mark Lawson (Heriot-Watt). In particular, it is expected that Dr. Lawson will visit my research group at the University of Ottawa, Canada, in Spring, 2012. Moreover, I hope to continue research contacts with the postgraduate students in LFCS I have been talking to during this visit.